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WAL 313 48

Interim Technical Report No. 2

February 1956

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RESIDUAL STRESSES IN CYLINDERS

Prepared by

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WATERTOWN ARSENAL

Contract No. DAI-3 [illegible] ORD [illegible]

Department of the Army, Project No. [illegible]

Ordnance Project No. [illegible]

SYRACUSE UNIVERSITY RESEARCH INSTITUTE

CHEMICAL AND METALLURGICAL ENGINEERING

Report No. MET3-F 563T2

WAL 313/48
Interim Technical Report No. 2

RESIDUAL STRESSES IN CYLINDERS

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February 1956

Work Performed under
Technical Supervision of
Watertown Arsenal Laboratory

Rochester Ordnance District
Contract No. DAI-30-115-505-ORD-(P)-613
Department of the Army Project No. 5B93-26-006
Ordnance Project No. TB4-911
Account No. 1620.345

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MET345-563T2

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ABSTRACT

The equations for residual-stress determinations on solid and hollow cylinders are derived from the force-equilibrium principle for the boring-out and the turning-off method. It is shown that the new equations are equivalent to the equations first derived by G. Sachs. However, the new equations offer considerable advantages because they can readily be solved by graphical computation methods. The graphical stress determination is described and an example is given.

ACKNOWLEDGEMENT

This work has been performed under contract with Watertown Arsenal, Contract No. DAI-30-115-505-ORD-(P)-613. The author is indebted to Dr. G. Sachs, Syracuse University, for the suggestion to investigate this problem and for his guidance in the work. He also wishes to express his gratitude to Dr. E. P. Klier and to Dr. K. N. Tong, Syracuse University, for their helpful cooperation. Furthermore, he is indebted to Dr. R. Beeuwkes, Chief Scientist, Ordnance Materials Research, and Project Supervisor on the above contract for making available his unpublished graphical method for stress determinations.

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INTRODUCTION

The boring-out method developed by Sachs (1) and its further development to the boring-out-turning-off method (2)(3)(4) are the only means of determining the complete residual stress pattern in specimens having cylindrical symmetry. Among the deficiencies of this method are a) an uncertainty of the signs of the stress components, b) the rather elaborate calculations of the stresses which also depend upon the subjective evaluation of the strain measurements, and c) the tedious and time-consuming measurements required to obtain accurate data. Nevertheless, these methods have been used extensively and by many investigators to obtain practically significant information (5).

However, recent studies by Buehler (6) and others (7) suggest certain modifications which promise considerable improvements and simplifications of the theory and particularly the practical applications of these methods.

Furthermore, Beeuwkes (8) has shown that the Sachs equations can be derived from the equilibrium of forces and replaced by simplified formulae which permit graphical computation of the stresses from strain functions with a minimum of effort.

In this report the Sachs equations and the corresponding equations for the combined boring-out-turning-off are derived following much the same reasoning as Beeuwkes. An attempt has been made to put the equations in a form which permits graphical determination of the stress distribution by a direct and accurate approach.

TERMINOLOGY

σ_ℓ longitudinal stress

σ_t tangential stress

σ_r radial stress

a inside radius, also index for inside surface of cylinder

b outside radius, also index for outside surface of cylinder

r instantaneous radius

$f_a = a^2\pi$

$f_b = b^2\pi$

$f = r^2\pi$

ε_ℓ measured longitudinal strain at either outside or inside surface

ε_t measured tangential strain at either outside or inside surface

$\lambda = \varepsilon_\ell + \mu\varepsilon_t$

$\theta = \varepsilon_t + \mu\varepsilon_\ell$

E modulus of elasticity

μ Poisson's ratio

$E' = \dfrac{E}{1-\mu^2}$

Sign convention: 1) stresses are positive for tension

2) df and dr positive

with increasing f and r for the boring-out process and with decreasing f and r for the turning-off method.*

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*This sign convention is equivalent with Sachs' sign convention; however, inconsistent with Buehler's, who had df, dr positive with increasing f, r for both processes.

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DERIVATION OF THE STRESS EQUATIONS

General

The new approach to the derivation of the residual-stress equations utilizes the equations for the equilibrium of forces which must be fulfilled by a body free from external loads, namely

$$\int_{f_a}^{f_b} \sigma_\ell \, df = 0 \qquad (1)$$

$$\int_{a}^{b} \sigma_t \, dr = 0 \qquad (2)$$

The longitudinal stresses, σ_ℓ , must balance out over the cross-sectional area, $f_b - f_a$, and the tangential stresses, σ_t , over the longitudinal section or radius, b - a.

The forces in one part of the body, therefore, can be replaced by those in another:

$$\int_{f_a}^{f} \sigma_\ell \, df + \int_{f}^{f_b} \sigma_\ell \, df = 0 \qquad (3)$$

$$\int_{a}^{r} \sigma_t \, dr + \int_{r}^{b} \sigma_t \, dr = 0 \qquad (4)$$

The change in stress in the remaining section of the bar due to the removal of a center core containing longitudinal, tangential and radial stresses can be expressed in terms of the strains measured on the outside surface by means of the generalized Hooke's Law:

$$\sigma_{\ell,b} = \frac{E}{1-\mu^2}\left(\varepsilon_\ell + \mu \cdot \varepsilon_t\right) = E'\lambda \qquad (5)$$

$$\sigma_{t,b} = \frac{E}{1-\mu^2}\left(\varepsilon_t + \mu \varepsilon_\ell\right) = E'\theta \qquad (6)$$

Longitudinal Stress

If the core of the cylinder from f_a to f is removed, the force removed is given by

$$P = \int_{f_a}^{f} \sigma_\ell \, df \tag{7}$$

This causes a uniform change of the stress distribution in the remaining section, i.e. also a change of the surface stresses to $\sigma_\ell + \sigma_\ell'$ where is a constant. The stresses must again be in equilibrium, i.e.

$$\int_{f}^{f_t} (\sigma_\ell + \sigma_\ell')\, df = \int_{f}^{f_t} \sigma_\ell \, df + \sigma_\ell' (f_t - f) = 0 \tag{8}$$

Replacing the first term of equation (8) and using equations (3) and (5) yields

$$\int_{f_a}^{f} \sigma_\ell \, df = E' \lambda_t (f_t - f) \tag{9}$$

The longitudinal stress removed is then obtained by differentiating equation (9) which yields

$$\sigma_\ell = E' \frac{d}{df} \left[\lambda_t (f_t - f) \right]^{*} \tag{10}$$

Completing the differentiation leads to the well-known Sachs equation, namely

$$\sigma_\ell = E' \left[(f_t - f) \frac{d\lambda_t}{df} - \lambda_t \right] \tag{11}$$

For practical stress determinations equation (10) represents a simplification compared to equation (11) because the longitudinal stress at a point "f" is now given by the derivative of a single quantity, or by the slope of the curve

*This equation is equivalent to that developed by Beeuwkes (8): $\sigma_\ell = E' \frac{1}{r} \frac{d}{dr} [\lambda_t (r_t^2 - r^2)]$
However, Beeuwkes' equation leads to a more complicated method of graphical computation.

$E'\lambda_b(f_b - f)$ vs. f, and no further calculations are necessary.

In the turning-off process metal is removed from the outside, the inside radius being a. The removed force is again given by $\int_{f_b}^{f} \sigma_l \, df$ which is distributed over the remaining section $(f - f_a)$. Equivalent to equation (9) the force relation for the turning-off process is

$$\int_{f_b}^{f} \sigma_l \, df = E'\lambda_a (f - f_a) \tag{12}$$

Differentiation of equation (12) gives the longitudinal stress for the turning-off process

$$\sigma_l = E' \frac{d}{df} \left[\lambda_a (f - f_a)\right] \tag{13}$$

which again is equivalent to the corresponding equation first given by Sachs and Espey (2) and later derived and proved by Buehler (3) and Hanslip (4)

$$\sigma_l = E' \left[(f - f_a) \frac{d\lambda_a}{df} + \lambda_a\right] \tag{14}$$

If a boring-out process has preceded the turning-off process, the force relieved by the boring operation, i.e.

$$\int_{0}^{f_a} \sigma_l \, df = E'\lambda_{b,a} (f_b - f) \tag{15}$$

has to be subtracted. In equation (15) $\lambda_{b,a}$ is the strain value obtained on the outside after boring was completed. The complete longitudinal stress for the combined boring-out-turning-off process is thus given by

$$\sigma_l = E' \frac{d}{df} \left[\lambda_a (f - f_a) - \lambda_{b,a}\right] \tag{16}$$

Tangential and Radial Stresses

The equations for the tangential and radial stresses can also be derived from a force equilibrium principle. Removing a cylindrical core by boring is equivalent to removing an internal pressure p which acted on the surface while the body is still intact. For the inside portion the equilibrium equation

$$p\,r = \int_a^r \sigma_t \, dr \tag{17}$$

applies, which is derived similar to equation (9) and using the relation

$$\frac{d(r \cdot \sigma_r)}{dr} = \sigma_t \tag{18}$$

For the outside portion, the tangential stress on the surface $\sigma_{t,b}$ is related to p by the formula, using also the equation (6)

$$\sigma_{t,b} = \frac{2\,r^2}{b^2 - r^2}\, p_r = E'\Theta_b \tag{19}$$

Consequently

$$p_r = \frac{b^2 - r^2}{2\,r^2}\, E'\,\Theta_b \tag{20}$$

and

$$\int_a^r \sigma_t \, dr = \frac{b^2 - r^2}{2\,r}\, E'\,\Theta_b \tag{21}$$

Differentiation of equation (21) leads to the equation for the tangential stress as determined by the boring-out method when the strain is measured on the outside surface, namely

$$\sigma_t = E' \frac{d}{dr}\left[\Theta_b \left(\frac{b^2 - r^2}{2r}\right)\right] \tag{22}$$

The radial stress is then obtained by using equation (18) and performing the integration resulting in*

$$\sigma_r = E'\theta_b \frac{b^2 - r^2}{2r^2} \tag{23}$$

Equations (22) and (23) can easily be shown to be equivalent to the corresponding Sachs equations by completing differentiation and replacing the radii with their corresponding f-values, namely

$$\sigma_t = E'\left[(f_b - f)\frac{d\theta_b}{df} - \theta_b \frac{f_b + f}{2f}\right] \tag{24}$$

and

$$\sigma_r = E'\theta_b \frac{f_b - f}{2f} \tag{25}$$

It is, however, convenient to use r and b rather than f-values in these equations because of the less complicated graphical evaluation. If f is used as an independent variable, σ_t is given by

$$\sigma_t = E'\sqrt{f}\frac{d}{df}\left[\theta_b\left(\frac{f_b - f}{\sqrt{f}}\right)\right] \tag{26}$$

For the turning-off process equation (21) is replaced by

$$\int_b^r \sigma_t \, dr = E'\theta_a \frac{r^2 - a^2}{2r} \tag{27}$$

Differentiation leads to the desired equation:

$$\sigma_t = E'\frac{d}{dr}\left[\theta_a\left(\frac{r^2 - a^2}{2r}\right)\right] \tag{28}$$

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*Beeuwkes (8) calculated first σ_r from equation (23) or (25) and then determined either σ_t or $(\sigma_t - \sigma_r)$ graphically, see equation (18): $\sigma_t - \sigma_r = \frac{d\sigma_r}{dr}$.

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Performing the integration according to equation (18) gives the radial stress component as

$$\sigma_r = E' \theta_a \frac{r^2 - a^2}{2r^2} \qquad (29)$$

During the preceding boring-out process tangential and radial stresses have been removed. This is equivalent to removing a pressure acting at the inside surface (radius = a) given by

$$p_a = \sigma_{r,a} = E' \theta_{b,a} \frac{b^2 - a^2}{2a^2} \qquad (30)$$

This pressure distributes itself according to

$$\sigma_r = E' \theta_{b,a} \frac{r^2 - b^2}{2r^2} \qquad (31)$$

which has to be subtracted from σ_r determined from equation (29), and thus gives the radial stress for the combined boring-out-turning-off process:

$$\sigma_r = \frac{E'}{2r^2} \left[\theta_a (r^2 - a^2) - \theta_{b,a} (r^2 - b^2) \right] \qquad (32)$$

where $\theta_{b,a}$ is the strain value obtained on the outside surface corresponding to an inside radius = a. Differentiation of equation (32) according to equation (18) gives

$$\sigma_t = E' \frac{d}{dr} \left\{ \frac{1}{2r} \left[\theta_a (r^2 - a^2) - \theta_{b,a} (r^2 - b^2) \right] \right\} \qquad (33)$$

which is the tangential stress component for the combined boring-out-turning-off method.

GRAPHICAL DETERMINATION OF STRESSES

With the new stress equations

$$\sigma_l = \frac{d}{df}\left[E'\lambda_b(f_b - f)\right]$$

$$\sigma_t = \frac{d}{dr}\left[E'\theta_b\left(\frac{b^2 - r^2}{2r}\right)\right]$$

$$\sigma_r = \frac{1}{r}\left[E'\theta_b\left(\frac{b^2 - r^2}{2r}\right)\right]$$

it is possible to use simple graphical methods for the computation of the stresses.

The longitudinal stress is constructed from a plot of $E'\lambda(f_b - f)$ vs. f by graphical slope measurements as illustrated in Fig. 1.

The tangential stress is determined in a similar way from a plot of $E'\theta\,\frac{b^2 - r^2}{2r}$ vs. r, as illustrated in Fig. 2.

From the same plot the radial stress is obtained as demonstrated in Fig. 3.

In constructing these graphs it is practical to select a power of 10 units of the area and radius respectively as unit of the abscissa and a power of 10 units of the strain function as ordinate. Then the stress becomes equal to the ordinate difference for the selected abscissa unit and thus can readily be transferred to a separate graph. It is convenient to determine these stresses for 0.5, 1.0, 1.5, etc. units of the abscissa, and, in addition, for a few selected values, including the boundaries.

Example of Stress Determination

The method is further illustrated for an actual case (9). The measured strain data are presented in Table I together with the quantities necessary

for constructing the two base curves. The two strain functions $E'\lambda_b(f_b - f)$ vs. f and $E'\vartheta_b \frac{b^2 - r^2}{2r}$ vs. r are represented in Figs. 4a and 5a. The stresses derived from these are shown in Figs. 4b and 5b. They are found to differ slightly from those derived in the reference (9) by the old method. It appears, however, that the graphical method yields slightly more accurate values particularly for the outer surface, as the strain functions for this position must smoothly approach zero.

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TABLE I

TEST RESULTS FROM REFERENCE (9)

Outside Radius $r_b = 117.05$ mm

Original Inside Radius $r_a = 39.035$ mm

$E' = 23.100$ kg/mm^2

$f_b = 43{,}040$ mm^2

$f_a = 4790$ mm^2

Measurement	r	f	$E'\lambda(f_b - f)$	$E'\theta\left(\frac{r_b^2 - r^2}{2r}\right)$
	mm	mm^2	10^4	10^2
0	39.035	4790	0	0
1	39.965	5020	-0.527	-0.140
2	45.185	6410	1.862	2.503
3	50.090	7890	4.466	3.719
4	51.260	8250	4.983	3.942
5	56.750	10110	9.966	4.919
6	60.930	11670	14.784	5.472
7	66.155	13790	20.136	6.023
8	70.025	15420	22.588	6.080
9	71.760	16200	24.368	6.125
10	75.225	17780	27.252	6.322
11	79.085	19650	29.558	6.177
12	82.620	21450	30.126	5.968
13	86.110	23300	30.965	5.598
14	88.375	24500	31.610	5.474
15	91.010	26100	30.213	5.012
16	93.225	27300	29.637	4.847
17	96.515	29600	26.207	4.067
18	100.295	31600	23.603	3.384
19	103.140	33400	20.669	2.902
20	105.315	34800	17.878	2.464
21	107.935	36500	14.749	1.960
22	109.865	37900	11.510	1.570

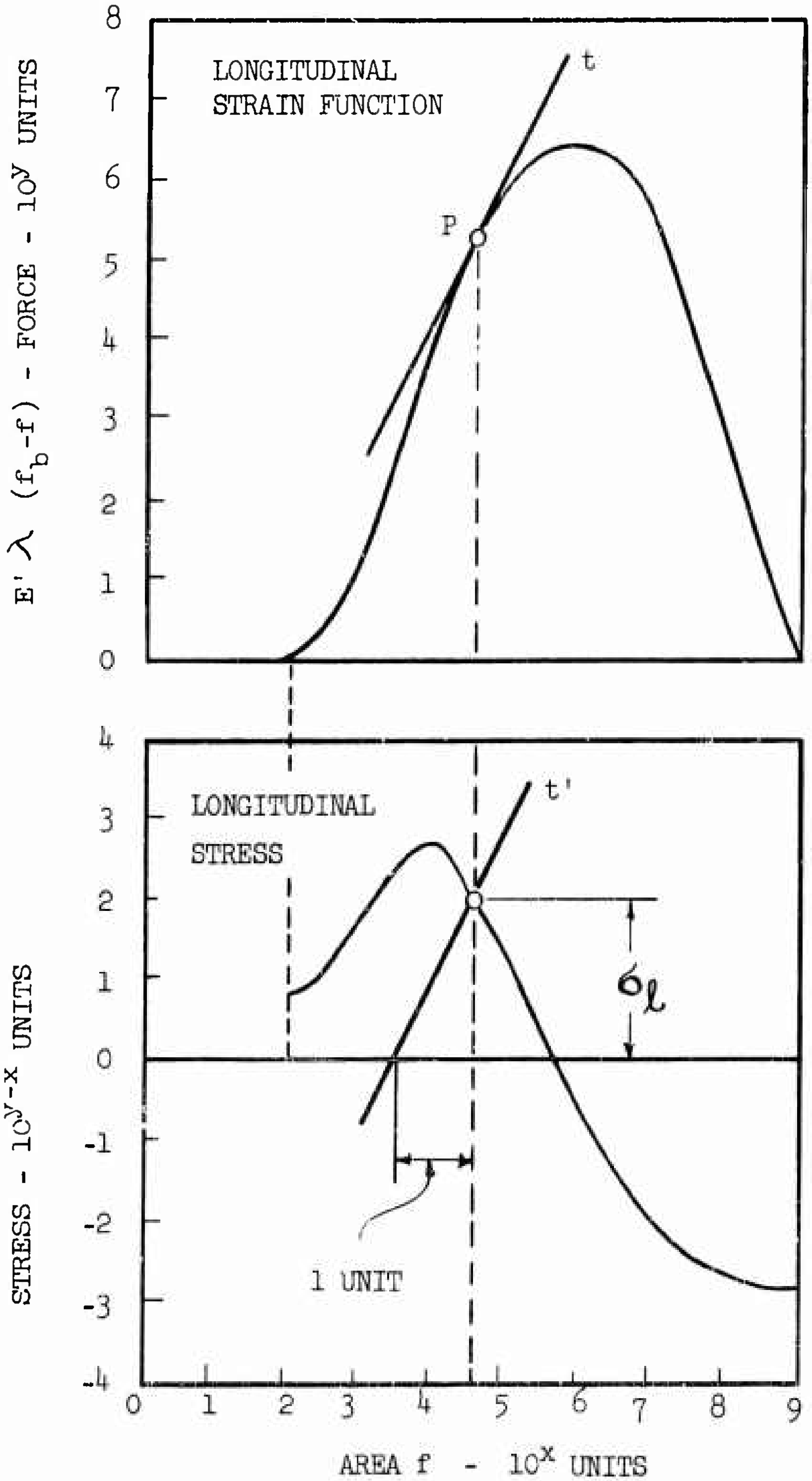

FIG. 1 DETERMINATION OF LONGITUDINAL STRESS

a.) PLOT $E'\lambda\ (f_b - f)$ vs. f

b.) DRAW TANGENT ON POINT P FOR WHICH THE STRESS IS TO BE DETERMINED.

c.) TRANSFER TANGENT t TO t' SO THAT IT INTERSECTS THE ABSCISSA AT THE POINT $(f-1).10^x$ UNITS.

d.) THE INTERSECTION OF t' AND THE ORDINATE THROUGH P GIVES THE DESIRED STRESS σ_ℓ .

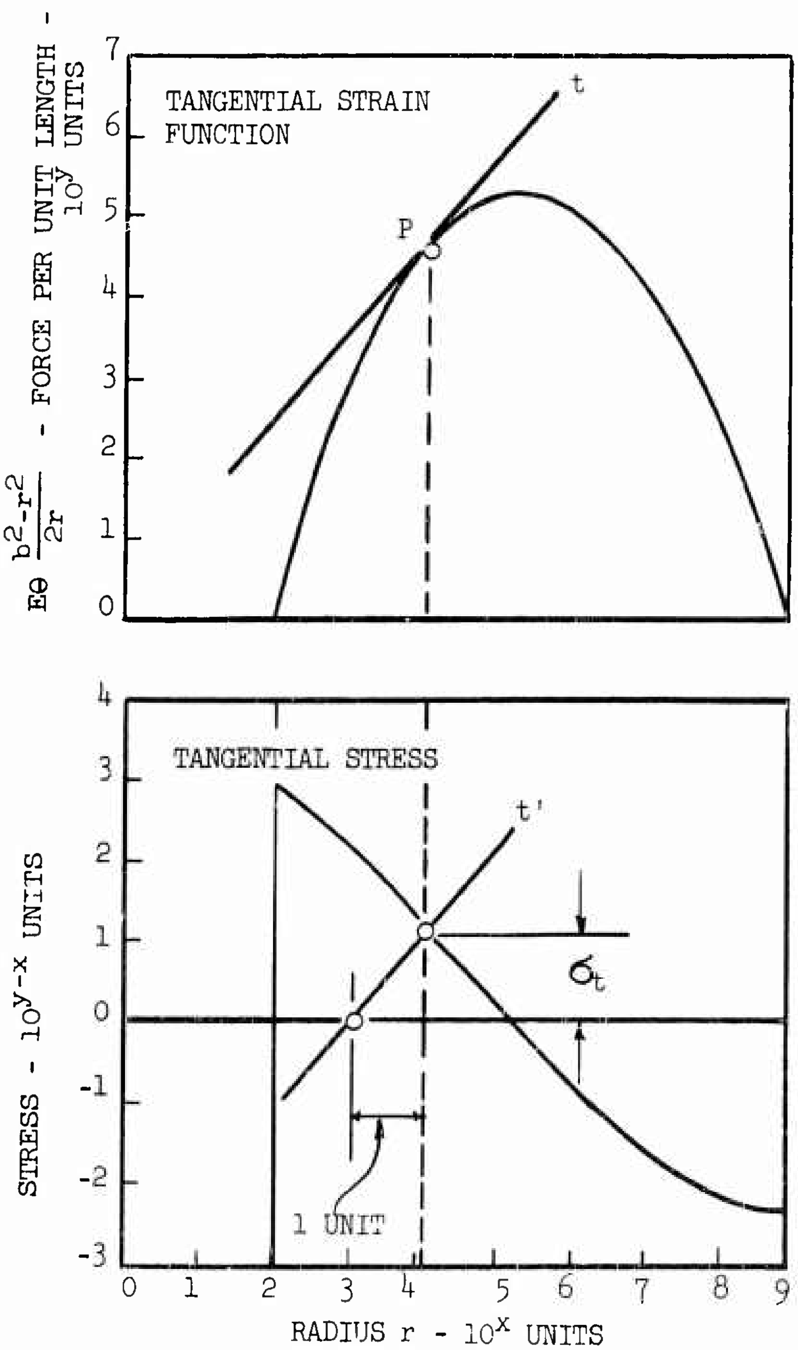

FIG. 2 DETERMINATION OF TANGENTIAL STRESS.

a.) PLOT $E'\Theta \frac{b^2-r^2}{2r}$ AGAINST RADIUS r.

b.) DRAW TANGENT t ON POINT P FOR WHICH THE STRESS IS TO BE DETERMINED .

c.) TRANSFER TANGENT t TO t' SO THAT IT INTERSECTS THE ABSCISSA AT THE POINT $(r-1)\cdot 10^x$ UNITS.

d.) THE INTERSECTION OF t' AND THE ORDINATE THROUGH P GIVES THE DESIRED STRESS σ_t.

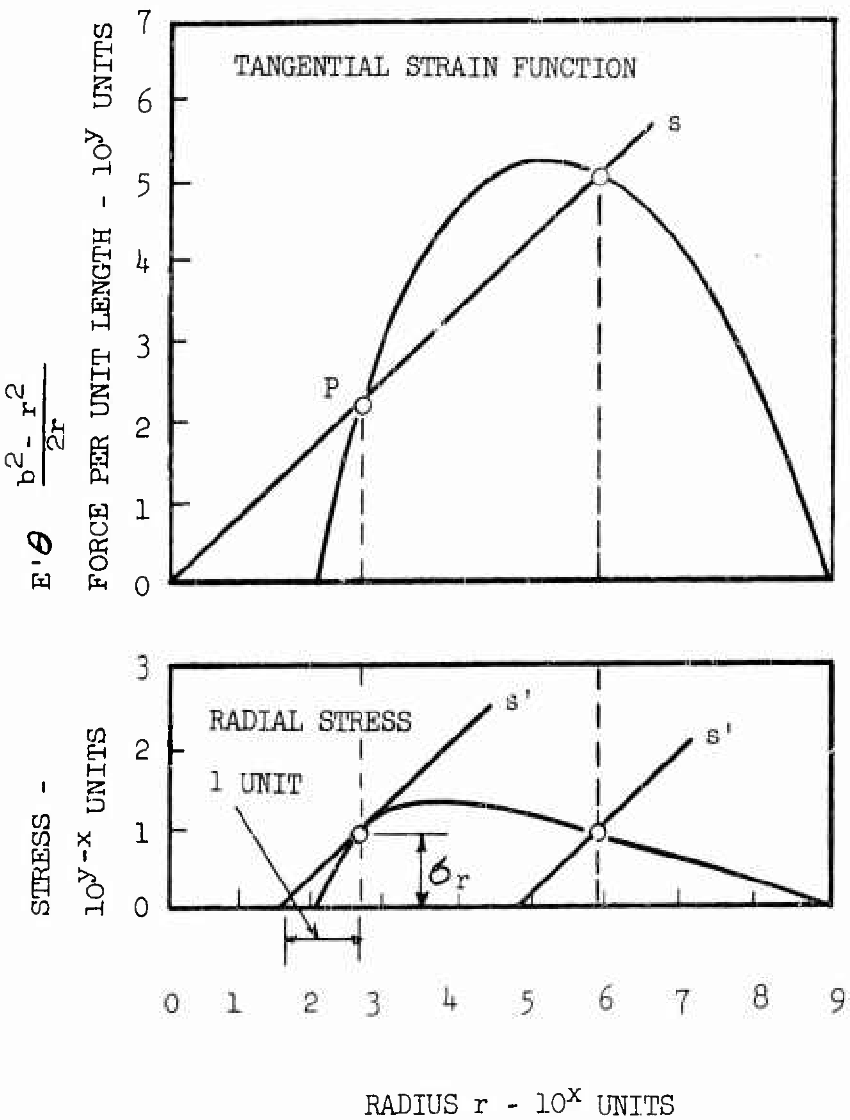

FIG. 3 DETERMINATION OF RADIAL STRESS

a.) PLOT $E'\theta \, \frac{b^2 - r^2}{2r}$ vs. r

b.) DRAW CONNECTION s THROUGH ORIGIN (0, 0) AND POINT P FOR WHICH STRESS HAS TO BE DETERMINED.

c.) TRANSFER s TO s' SO THAT IT INTERSECTS $\sigma = 0$ AT THE POINT $(r-1).10^x$ UNITS.

d.) THE INTERSECTION OF s' AND THE ORDINATE THROUGH P GIVES THE DESIRED STRESS σ_r.

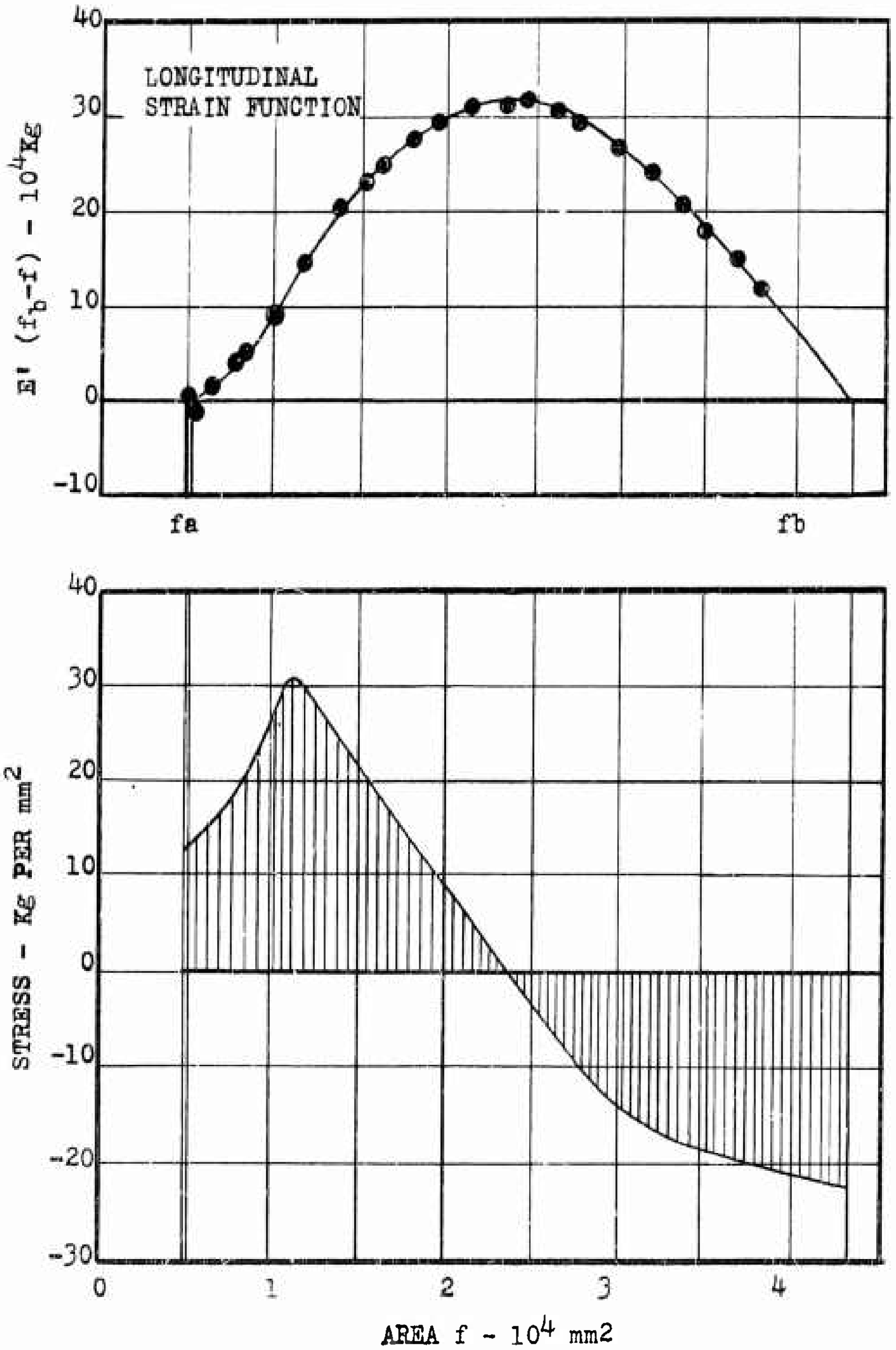

FIG. 4 LONGITUDINAL STRAIN FUNCTION AND LONGITUDINAL STRESS.

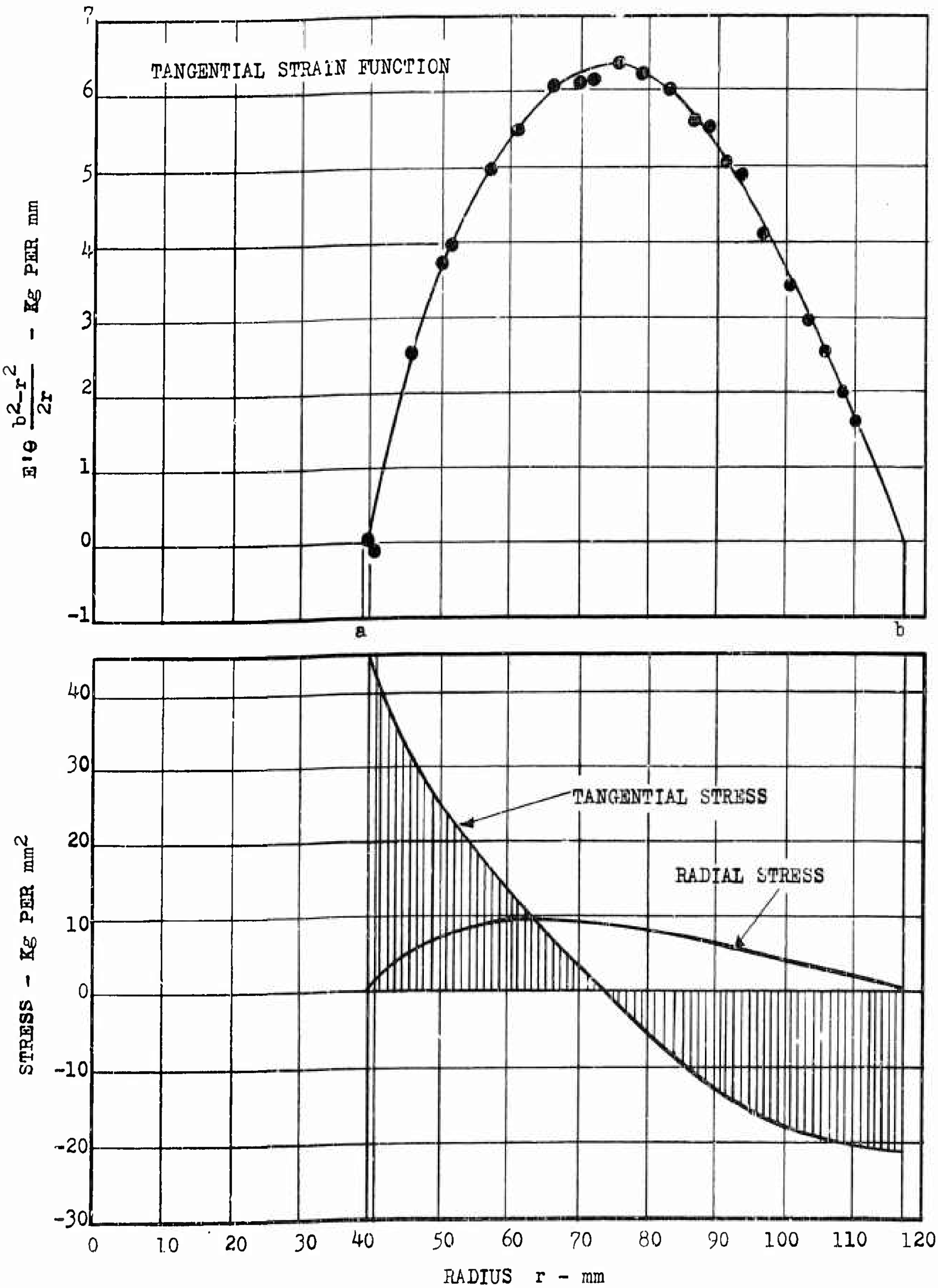

FIG. 5 TANGENTIAL STRAIN FUNCTION AND TANGENTIAL AND RADIAL STRESS.